AVERAGE EMISSIVITIES OF VARIOUS SHAPED VOLUMES

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A method is developed for determining the average emissivity of a volume, based on a comparison of volumes of various shapes with the same characteristic size. The selectivity of the radiation leads to a sharp decrease in the maximum error as compared with the calculation for monochromatic radiation.

Three methods for calculating average emissivities of various shaped volumes are described in [1]. Essentially they all use the fact that volumes with the same characteristic size $l_0 = 4V/F$ have nearly the same emissivities. We develop a comparison method in which the emissivity of a slab is taken as a standard. We investigate the dependence of the differences $\Delta\varepsilon_i = \varepsilon_i - \varepsilon_0$ on the size l_0, the nature of the radiation, and the temperature of the gas. The final aim of the studies is the creation of a convenient engineering method of calculation based on tables or graphs of ε_0 and graphs of the differences $\Delta\varepsilon_i$ constructed for volumes of simple shapes.

Figure 1 shows the effect of the shape of a body and the size $\alpha l_0/2$ on the differences $\Delta\varepsilon_i$ for monochromatic or gray radiation. Curves 1 and 2 were constructed long ago; curve 5 up to $\alpha l_0 = 2$ is in [1] (Fig. 101). The remaining curves and the part of curve 5 for $\alpha l_0 > 2$ appear here for the first time.

If we limit ourselves to the range of optical thicknesses where the differences $\Delta\varepsilon_i$ are important, the sphere represents the limiting case with the largest emissivity. The curves do not have definite lower bounds.

The parts of the surface close to the edges and corners play a special role. Rays emerging through these parts are mostly short. Therefore, their contribution to the total radiation becomes significant only for sufficiently large αl_0. The ratio of these parts to the total surface of a body increases in the sequence 2, 3, ..., 7. Consequently, the differences $|\Delta\varepsilon|_{max}$ having a negative sign increase also. A certain displacement of the minima is observed.

On the basis of the above considerations the following can be predicted. The curve for a bar with a triangular cross section passes below curve 5 and occupies the lowest position for bars whose cross sections are regular polyhedra. On the other hand, the curve for a bar with a regular pentagonal cross section passes above curve 5. For cross sections which are irregular polyhedra, for example, a wedge, the solution cannot be so definite. Similar considerations lead to the conclusion that the curve for a regular tetrahedron passes below the curve for a cube and occupies the limiting position among the curves for regular polyhedra.

It is considerably more complicated to predict the result for bodies with concave parts. In a number of cases complex forms can be built up from simple shapes and their emissivities can be determined by an elementary calculation. As an example, we consider a sphere with a washer around it in a diametral plane. The volumes of the bodies are assumed the same. The ratio of the diameter of the sphere to the thickness of the washer is 12. With negligible error the washer can be considered an infinite plane layer. The partial decrease of the surface of the sphere can be neglected. Then the average emissivity of the combination of the layer and the sphere can be determined in an elementary fashion. This case is shown by the open curve of Fig. 1. In this example the fraction of the portions of the surface with

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relatively short ray lengths is hypertrophied.

Figure 1 gives an estimate of the value of $\Delta\varepsilon_i$ after comparing the shape of the given body with the closest shape shown in the figure. The emissivity of the volume is

$$\varepsilon_i = \varepsilon_0 + \Delta\varepsilon_i, \ \varepsilon_0 = 1 - 2E_3(\alpha l_0/2).$$

In a turbid medium with isotropic scattering it is necessary to use the argument $\tau_0 = kl_0$. The effect of anisotropic scattering is not considered here. The following approximation is recommended for the exponential integral E_3:

$$E_3(x) = 0{,}0314e^{-8.8x} + 0{,}2222e^{-2x} + 0{,}2464e^{-1.125x}.$$

The approximate formula underestimates $E_3(x)$ by 0.3% for x = 0.1, by 0.34% for x = 1.8, and overestimates E_3 by 0.55% for x = 0.8. These are the maximum deviations in the interval $0 \le x \le 2$. For $x > 2$ the function is so small that the error in its evaluation is insignificant. In the most accurate calculations a table of values of ε_0 must be used.

Example. We determine the monochromatic emissivity averaged over the surface of a parallelepiped with sides: $\Delta = 1$ m, H = 2 m, L = 4 m, $\alpha = 1\ m^{-1}$. With these values $\alpha l_0/2 = 0.5715$.

Curves 6 and 7 of Fig. 1 are for bodies of nearly the same shape; a square bar with a length 10 times that of a side of its cross section and a cube. In relative length ($l = L/\Delta = 4$) the given body is in between bodies 6 and 7, but closer to body 6. The cross section of the body is flattened in comparison with bodies 6 and 7. In Fig. 1 this gives a further shift of the ordinate toward decreasing $\Delta\varepsilon$. Therefore we determine $\Delta\varepsilon = 0.006$ halfway between curves 6 and 7. We note that for $\alpha l_0/2 > 0.7$ the signs of the shifts with respect to curve 6 are different.

We determine ε_0 from a table of values of E_3: $\varepsilon_0 = 1 - 2E_3\ (0.5715) = 0.5995$. We obtain $\varepsilon_i = 0.5995 + 0.006 = 0.6055$.

The result is easily checked by the tables in [7]. We find the emissivity of each face from the expression

$$\varepsilon = 1 - \varphi_{AB} - 2\varphi'_{AC} - 2\varphi''_{AC}.$$

The quantities φ'_{AC} and φ''_{AC} here refer to the different lateral faces C. The emissivities of the faces are

$$2 \times 4, \ \varepsilon_1 = 0.6235;$$

$$1 \times 4, \ \varepsilon_2 = 0.5929;$$

$$1 \times 2, \ \varepsilon_3 = 0.5586.$$

We determine the average emissivity over all the faces from the expression

$$\varepsilon = (8\varepsilon_1 + 4\varepsilon_2 + 2\varepsilon_3)/14 = 0.6055.$$

The complete agreement of the results is fortuitous, since the tables of angular coefficients used nine times in the calculation have errors in the fourth decimal place.

The example shows clearly the simplicity and reliability of the method based on the use of the curves of Fig. 1.

Remarks on Fig. 1. Tables and approximate formulas of angular coefficients published in a series of papers by the authors were used to construct the new curves.

The values of ε_0 and ε_1 were calculated by exact formulas. Values of ε_2 are tabulated by various authors.

For a finite cylinder (curves 3 and 4)

$$\varepsilon_1 = 1 - [\varphi_{13} + 2h(\varphi_{22} + 4\varphi_{23})]/(1 + 2h).$$

Here the subscripts 1 and 3 denote the ends and subscript 2 denotes the lateral surface. We used the tables of angular coefficients in [3, 4] and approximate equations in [3].

For a square bar

$$\varepsilon_5 = 1 - \varphi_{AB} - 2\varphi_{AC}.$$

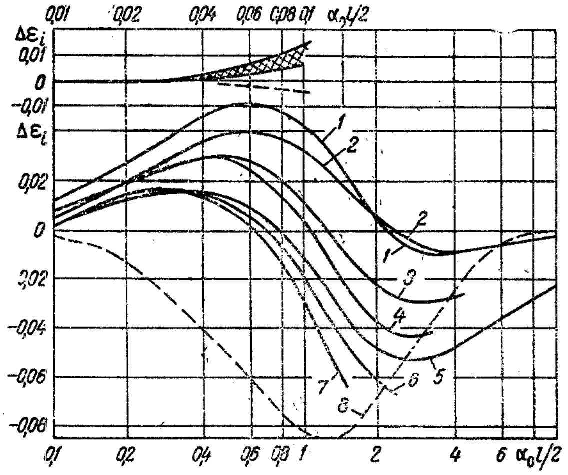

Fig. 1

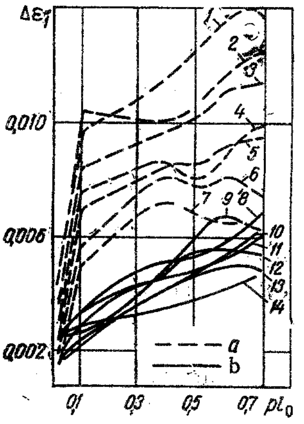

Fig. 2

Fig. 1. $\Delta\varepsilon_i = \varepsilon_i - \varepsilon_0$ as a function of the dimensionless characteristic size $\alpha l_0/2$ for monochromatic radiation: 1) sphere; 2) infinite circular cylinder; 3) circular cylinder with H/D = 3; 4) the same for H/D = 1; 5) infinitely long square bar; 6) square bar whose length is 10 times that of a side of its cross section; 7) cube; 8) combination body consisting of a sphere and a slab.

Fig. 2. Differences in emissivities of a sphere and a slab as a function of size pl_0 at various temperatures. 1) 600°C; 2) 800; 3) 1000; 4) 1200; 5) 1400; 6) 1600; 7) 1800; 8) 1000; 9) 1600; 10) 800; 11) 1200; 12) 1400; 13) 1800; 14) 600; a) curves for water vapor; b) curves for carbon dioxide; p, partial pressure; pl_0 in m·atm.

Subscripts B and C denote the sides of the bar parallel and perpendicular to its base A. Up to $\alpha l_0 = 2$ the values were found from tables in [5]. For $3.3 \le \alpha l_0 \le 10$ the approximate formulas in [6] were used.

For a band spectrum

$$\Delta\varepsilon_i = \frac{\pi}{\sigma T^4} \sum_j I_{0j} \Delta\varepsilon_{ij} \Delta\omega_j.$$

The values of $\Delta\varepsilon_{i\omega}$ and $\Delta\varepsilon_{ij}$ can be estimated from Fig. 1 if the spectral absorption coefficients or their values averaged over the band are known. Depending on the part of the spectrum the quantities $\Delta\varepsilon_{i\omega}$ have different values and can even change sign. Therefore the values of $\Delta\varepsilon_i$ integrated over the spectrum will always be smaller than the maximum deviations $\Delta\varepsilon_{i\omega}$ shown in Fig. 1. This general consideration was presented in [10]. Now it is illustrated by the example of carbon dioxide and water vapor (Fig. 2). Some of the data were taken from [11] but most of it is unpublished and was given us by A. S. Nevskii. The difference $\Delta\varepsilon_1$ is comparable in magnitude with the error in the values of ε_0 and ε_1 accumulated in a painstaking calculational process. Therefore, the curves in Fig. 2 are rough. The scale of ordinates is appreciably larger than in Fig. 1. The differences $\Delta\varepsilon$ obtained by taking account of the selectivity of the gases are almost an order of magnitude smaller. They decrease particularly sharply for carbon dioxide, giving a flux with a sharper change of the spectral coefficient than for water vapor. Unfortunately there are no data in the literature for other gases or for bodies of other shapes.

For a bar the positive differences $\Delta\varepsilon_{5\omega}$ will be completely compensated by the negative differences $\Delta\varepsilon_{5\omega}$. Therefore, one should expect the values of $\Delta\varepsilon_5$ integrated over the spectrum to be still smaller than the values of $\Delta\varepsilon_1$.

The emissivity of a layer of a real medium can be determined from the approximate expression [10]

$$\varepsilon_0 = 0.0628\varepsilon(8.8x) + 0.4444\varepsilon(2x) + 0.4928\varepsilon(1.125x).$$

Here $\varepsilon(8.8x)$ is the one-dimensional emissivity for the temperature of the medium and a thickness 8.8x.

The decrease in the differences $\Delta\varepsilon_i$ in the transition from monochromatic radiation to an actual spectrum greatly favors a wider use of the recommended method of calculation.

NOTATION

$l_0 = 4V/F$, characteristic size of volume; V, volume; F, surface of body; ε_0, emissivity of a slab; ε_i, average emissivities of other shaped volumes of the same size l_0; α, absorption coefficient, m^{-1}; k, attenuation coefficient, m^{-1}; E_3, third-order exponential integral; ω, wave number, cm^{-1}; λ, wavelength, μ; I_{0_ω}, Planck function, W cm/m·ster; I_{0_j}, the same for center of band j; φ, angular coefficient taking account of attenuation of radiation in the medium separating the surfaces; H, height of cylinder, m; D, diameter, m; L, length of parallelepiped, m; h = H/D; $l = L/\Delta$; Δ, base of cross section of body; $x = pl$; p, partial pressure of radiating gas.

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